Novel application of a perturbed photonic crystal: High-quality filter

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The transmission of light waves through a perturbed photonic crystal has been investigated. The perturbed photonic crystal was constructed by randomly repeated stacking of a number of identical unit cells. Although most of the light waves are localized by the randomness, there still exist light waves with special wavelengths which are in extended states. This produces a high-quality resonant tunneling with a very narrow transmission coefficient peak. © *1997 American Institute of Physics.* [S0003-6951(97)01246-1]

The concept of photonic crystal stems from early ideas of Yablonovitch and John.[1] The idea is to design materials so that they can affect the properties of photons, in much the same way that ordinary semiconductor crystals affect the properties of electrons. In particular, photonic crystals forbid propagation of photons having a certain range of energies (known as photonic band gaps), which could be incorporated into the design of novel optoelectronic devices.[2] Following the demonstration of a material with full photonic band gap at microwave frequencies,[3] there has been considerable progress in the fabrication of three-dimensional photonic crystals with operational wavelength as short as 1.5 μm.[4] Although most of the efforts focused on the search of a material that exhibits a full photonic band gap, it has been recognized that the introduction of defects into the photonic crystal, either locally or in an extended region, will allow us to generate electromagnetic states with specific properties.[5–12] In this letter, we study light transmission in a type of perturbed photonic crystal. Here we employ one-dimensional photonic crystal[13,14] as a simple example. This type of perturbed photonic crystal is shown in Fig. 1. The motion of light in such a perturbed photonic crystal is analogous to the motion along a one-dimensional disordered system. According to the scaling theory,[15] all the wave functions are localized in such a system. However, for electrons in a one-dimensional random double barrier system, Dunlap *et al.*[16] have found a small amount of states which are still extended under this type of randomness. On the basis of these findings we think it may be interesting to investigate the transmission of light through a perturbed photonic crystal. The results show that it is possible to produce a high-quality resonant tunneling with a very narrow transmission peak. This is a novel mechanism for an optical filter.

The thickness of the ith spacer d_i, satisfies a uniform probability distribution

$$P(d_i)=\begin{cases}1/(d_2-d_1) & \text{for } d_2\geqslant d_i\geqslant d_1,\\ 0, & \text{otherwise}.\end{cases} \tag{1}$$

The transmission of light through one unit cell (A and B) can be represented by the characteristic matrix $T_\wedge$[17]

$$T_\wedge=\begin{pmatrix}\cos\delta_A\cos\delta_B-\dfrac{n_B}{n_A}\sin\delta_A\sin\delta_B & -\dfrac{i}{n_B}\cos\delta_A\sin\delta_B-\dfrac{i}{n_A}\sin\delta_A\cos\delta_B\\ -in_A\sin\delta_A\cos\delta_B-in_B\cos\delta_A\sin\delta_B & \cos\delta_A\cos\delta_B-\dfrac{n_A}{n_B}\sin\delta_A\sin\delta_B\end{pmatrix} \tag{2}$$

in which the phase δ is given by $\delta_{A(B)}=n_{A(B)}kd_{A(B)}$, where k is the wave vector in vacuum and where $d_{A(B)}$ is the thickness of the components. The transmission of a lightwave through the above mentioned perturbed photonic crystal is represented by a matrix string

$$T=T_MT_{S1}T_MT_{S2}\dots T_MT_{Si}\dots T_MT_{SN}T_M, \tag{3}$$

where T_M is the transfer matrix for M unit cells, T_{S_i} ($i=1,2,...,N$) is the matrix for the ith spacer. Thus, the transmission coefficient for tunneling through such a structure can be calculated as

$$t=\frac{4}{|T_{11}+T_{22}|^2+|T_{12}+T_{21}|^2}, \tag{4}$$

where T_{ij} are the elements of the matrix T.

T_M can be further decomposed, $T_M=(T_\wedge)^M$. From the theory of matrices, the Mth power of the 2×2 unimodular matrix $T_\wedge$ can be written as[17]

$$T_M=\mu_{M-1}(x)T_\wedge-\mu_{M-2}(x)I, \tag{5}$$

where $x=\frac{1}{2}\,\mathrm{Tr}(T_\wedge)$, and $\mu_M(x)$ is the Chebyshev polynomial of the second kind, which obeys the recurrence relation

$$\mu_{M+1}(x)=2x\mu_M(x)-\mu_{M-1}(x)I,\quad M\geqslant 0 \tag{6}$$

with $\mu_{-1}=0$, $\mu_0=1$, and

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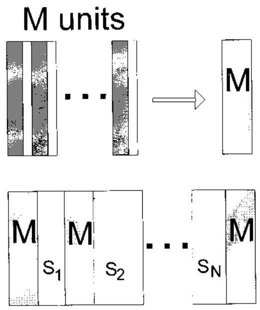

FIG. 1. The perturbed photonic crystal is constructed by repeated stacking of a number of identical unit cells, each of which is an alternating array of *M* identical high-dielectric components and *M* identical low-dielectric components. These unit cells are separated by *N* spacers, which are of the same material as the low-dielectric component, but the thickness of the spacers is a random variable.

$$\mu_{M-1}(x)=\frac{\sin[M\ \cos^{-1}(x)]}{\sin[\cos^{-1}(x)]}. \tag{7}$$

For the spacers

$$T_{s_i}=\begin{pmatrix} \cos\ \delta_i & -\dfrac{i}{n_A}\sin\ \delta_i \\ -in_A\ \sin\ \delta_i & \cos\ \delta_i \end{pmatrix}, \tag{8}$$

where $\delta_i=n_ikd_i$, d_i is the thickness of the ith spacer. If

$$x=\cos\left(\frac{l\pi}{M}\right)\equiv x_l,\quad l=1,2,...,M-1,\quad \text{for } M\geqslant 2, \tag{9}$$

we have $\mu_{M-1}(x_l)=0$, and $\mu_{M-2}(x_l)=(-1)^{l+1}$. From Eq. (5), we have $T_M=(-1)^lI$, and

$$T_s=\begin{pmatrix} \cos\ \delta & -\dfrac{i}{n_A}\sin\ \delta \\ -in_A\ \sin\ \delta & \cos\ \delta \end{pmatrix}, \tag{10}$$

where $\delta=n_AkD$, D is the total length of all spacers.

Under the condition of Eq. (9), the transmission coefficient is equivalent to that of a lightwave through a homogenous region of low-dielectric medium with thickness D. This means that the states with wavelength λ satisfying Eq. (9) are completely unscattered. We iterate numerically the matrices of Eq. (3). Figure 2 shows the calculated result of the transmission–wavelength curves. It is striking to note that most states are localized due to the randomness, the transmission coefficient for these states vanishes, but there still exist states with certain wavelengths which have high-quality tunneling with very narrow transmission peaks.

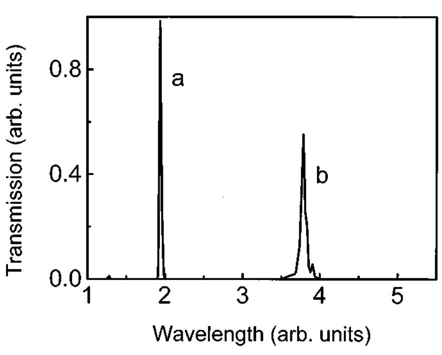

FIG. 2. The transmission coefficient as a function of incident wavelength. The parameters: $n_A/n_B=3.4$, $d_A/d_B=0.286$, $M=2$, $(d_2-d_1)/d_B=1$, and $N=50$.

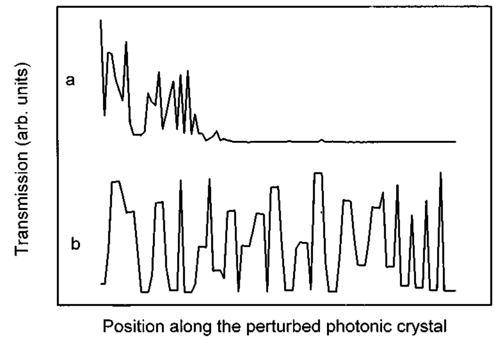

FIG. 3. Light transmission coefficient as a function of position in the perturbed one-dimensional photonic crystal. The parameters are the same as Fig. 2. (a) λ=4.5, (b) λ=2.0.

In order to investigate directly the physical reason for this phenomenon, we study the light amplitude as a function of position in such a photonic crystal. Most injected lightwaves are localized in the photonic crystal, such as light of λ=4.5, which was in a passband in an ordinary photonic crystal with $n_A/n_B=3.4$ and $d_A/d_B=0.286$. When the photonic crystal is perturbed, it would be localized as shown in Fig. 3(a). But for lightwaves with certain wavelength, its amplitude is Bloch wavelike and is clearly delocalized (or extended) [see Fig. 3(b)]. It is fundamental to note that both localization and delocalization phenomena are pure consequences of Maxwell's equations and disorder. In other words, the phenomena are observed numerically, but no condition of localization or delocalization has been introduced into the theory.

For the application of this novel phenomenon, another aspect is the determination of the locations of the resonant tunneling states. In fact, from Eq. (9) we have

$$\cos\ \delta_A^*\ \cos\ \delta_B^*-\left(\frac{n_A}{n_B}+\frac{n_B}{n_A}\right)\sin\ \delta_A^*\ \sin\ \delta_B^*=\cos\left(\frac{l\pi}{M}\right),$$
$$l=1,2,...,M-1, \tag{11}$$

where $\delta_A^*=n_Ak_ld_A$, $\delta_B^*=n_Bk_ld_B$, and k_l is the corresponding wave vectors. For lightwave with wave vector k_l, the transfer matrix string of the perturbed photonic crystal is only composed of matrix T_s and $(-1)^lI$. This matrix string is equivalent to that of the homogeneous medium of the spacers with thickness of $D=\Sigma d_i$. For this perturbed photonic crystal, the light with wave vector k_l is an extended state, which is completely unscattered. The wavelength locations of the extended states also can be predicted from Eq. (11). Figure 4 depicts the wavelength locations which change with the dielectric constant contrast of the perturbed photonic crystal for fixed structural parameters. It can be found that the tunneling wavelength is sensitive to n_A/n_B.

In conclusion, we have shown that for a kind of perturbed photonic crystal, although most states are localized due to the randomness, there exist extended states with certain wavelengths. This produces a high-quality resonant tunneling with a transmission peak much narrower than that of the tunneling through a perfect photonic crystal. By using

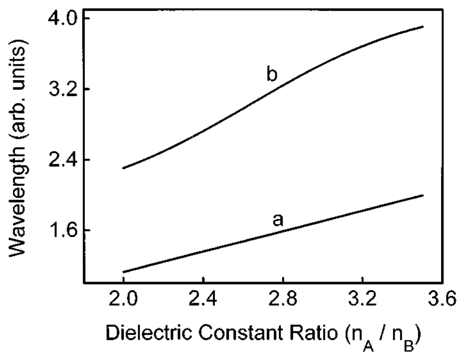

FIG. 4. Wavelength locations of unscattered states for samples with $d_A/d_B=0.286$, $M=2$, $(d_2-d_1)/d_B=1$, and $N=50$. Here a and b curves represent peak positions a and b in Fig. 2, respectively.

this kind of photonic microstructure in an appropriate material system, the optical filtering could be adapted to the delocalization property. We believe that these results are significant towards the realization of novel optoelectronic devices based on the new concept of photonic crystals.

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