The concept of photonic crystal stems from early ideas of Yablonovitch and John.\textsuperscript{1} The idea is to design materials so that they can affect the properties of photons, in much the same way that ordinary semiconductor crystals affect the properties of electrons. In particular, photonic crystals forbid propagation of photons having a certain range of energies (known as photonic band gaps), which could be incorporated into the design of novel optoelectronic devices.\textsuperscript{2} Following the demonstration of a material with full photonic band gap at microwave frequencies,\textsuperscript{3} there has been considerable progress in the fabrication of three-dimensional photonic crystals with operational wavelength as short as 1.5 $\mu$m.\textsuperscript{4} Although most of the efforts focused on the search of a material that exhibits a full photonic band gap, it has been recognized that the introduction of defects into the photonic crystal, either locally or in an extended region, will allow us to generate electromagnetic states with specific properties.\textsuperscript{5–12} In this letter, we study light transmission in a type of perturbed photonic crystal. Here we employ one-dimensional photonic crystal\textsuperscript{13,14} as a simple example. This type of perturbed photonic crystal is shown in Fig. 1. The motion of light in such a perturbed photonic crystal is analogous to the motion along a one-dimensional disordered system. According to the scaling theory,\textsuperscript{15} all the wave functions are localized in such a system. However, for electrons in a one-dimensional random double barrier system, Dunlap et al.\textsuperscript{16} have found a small amount of states which are still extended under this type of randomness. On the basis of these findings we think it may be interesting to investigate the transmission of light through a perturbed photonic crystal. The results show that it is possible to produce a high-quality resonant tunneling with a very narrow transmission peak. This is a novel mechanism for an optical filter.

The thickness of the $i$th spacer $d_i$, satisfies a uniform probability distribution

$$P(d_i) = \begin{cases} \frac{1}{l}(d_2 - d_1) & \text{for } d_2 \geq d_i \geq d_1, \\ 0, & \text{otherwise}. \end{cases}$$

The transmission of light through one unit cell ($A$ and $B$) can be represented by the characteristic matrix $T_A$.\textsuperscript{17}

$$T_A = \begin{pmatrix} \cos \delta_A \cos \delta_B - \frac{n_B}{n_A} \sin \delta_A \sin \delta_B & -i \frac{\cos \delta_A \sin \delta_B - i \frac{n_B}{n_A} \sin \delta_A \cos \delta_B}{2} \\ -i n_A \sin \delta_A \cos \delta_B - i n_B \cos \delta_A \sin \delta_B & \cos \delta_A \cos \delta_B - \frac{n_A}{n_B} \sin \delta_A \sin \delta_B \end{pmatrix}$$

in which the phase $\delta$ is given by $\delta_{A(B)} = \frac{n_{A(B)} k d_{A(B)}}{2}$, where $k$ is the wave vector in vacuum and where $d_{A(B)}$ is the thickness of the components. The transmission of a lightwave through the above mentioned perturbed photonic crystal is represented by a matrix string

$$T = T_M T_S_1 T_M T_S_2 \cdots T_M T_S_1 \cdots T_M T_S_N T_M,$$  

where $T_M$ is the transfer matrix for $M$ unit cells, $T_S_i$ ($i = 1,2,\ldots,N$) is the matrix for the $i$th spacer. Thus, the transmission coefficient for tunneling through such a structure can be calculated as

$$t = \frac{4}{|T_{11} + T_{22}|^2 + |T_{12} + T_{21}|^2},$$

where $T_{ij}$ are the elements of the matrix $T$.

$T_M$ can be further decomposed, $T_M = (T_\Lambda)^M$. From the theory of matrices, the $M$th power of the $2 \times 2$ unimodular matrix $T_\Lambda$ can be written as\textsuperscript{17}

$$T_M = \mu_{M-1}(x) T_\Lambda - \mu_{M-2}(x) I,$$

where $x = \frac{1}{2} \text{Tr}(T_\Lambda)$, and $\mu_m(x)$ is the Chebyshev polynomial of the second kind, which obeys the recurrence relation

$$\mu_{M+1}(x) = 2x \mu_M(x) - \mu_{M-1}(x), \quad M \geq 0$$

with $\mu_{-1} = 0, \mu_0 = 1, \text{ and}$

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that most states are localized due to the randomness, the transmission–wavelength curves. It is striking to note that the tunneling wavelength is sensitive to some states, which are completely unscattered. The wavelength locations which change with the dielectric constant contrast of the perturbed photonic crystal for fixed structural parameters. It can be found that the tunneling wavelength is sensitive to $n_A/n_B$.

In conclusion, we have shown that for a kind of perturbed photonic crystal, although most states are localized due to the randomness, there exist extended states with certain wavelengths which have high-quality tunneling with very narrow transmission peaks.

In order to investigate directly the physical reason for this phenomenon, we study the light amplitude as a function of position in such a photonic crystal. Most injected lightwaves are localized in the photonic crystal, such as light of $\lambda=4.5$, which was in a passband in an ordinary photonic crystal with $n_A/n_B=3.4$ and $d_A/d_B=0.286$. When the photonic crystal is perturbed, it would be localized as shown in Fig. 3(a). But for lightwaves with certain wavelength, its amplitude is Bloch wave-like and is clearly delocalized (or extended) [see Fig. 3(b)]. It is fundamental to note that both localization and delocalization phenomena are pure consequences of Maxwell’s equations and disorder. In other words, the phenomena are observed numerically, but no condition of localization or delocalization has been introduced into the theory.

For the application of this novel phenomenon, another aspect is the determination of the locations of the resonant tunneling states. In fact, from Eq. (9) we have

$$\cos \delta_l^* \cos \delta_l^* - \left( \frac{n_A}{n_B} + \frac{n_B}{n_A} \right) \sin \delta_l^* \sin \delta_l^* = \cos \left( \frac{l \pi}{M} \right),$$

$$l=1,2,\ldots,M-1,$$

where $\delta_l^* = n_A k_l d_A$, $\delta_l^* = n_B k_l d_B$, and $k_l$ is the corresponding wave vectors. For lightwave with wave vector $k_l$, the transfer matrix string of the perturbed photonic crystal is only composed of matrix $T_l$ and $(-1)^{l+1} I$. This matrix string is equivalent to that of the homogeneous medium of the spacers with thickness of $D=\Sigma d_i$. For this perturbed photonic crystal, the light with wave vector $k_l$ is an extended state, which is completely unscattered. The wavelength locations of the extended states also can be predicted from Eq. (11). Figure 4 depicts the wavelength locations which change with the dielectric constant contrast of the perturbed photonic crystal for fixed structural parameters. It can be found that the tunneling wavelength is sensitive to $n_A/n_B$.

In conclusion, we have shown that for a kind of perturbed photonic crystal, although most states are localized due to the randomness, there exist extended states with certain wavelengths. This produces a high-quality resonant tunneling with a transmission peak much narrower than that of the tunneling through a perfect photonic crystal. By using
FIG. 4. Wavelength locations of unscattered states for samples with \( d_A/d_B = 0.286 \), \( M = 2 \), \( (d_2-d_1)/d_B = 1 \), and \( N = 50 \). Here \( a \) and \( b \) curves represent peak positions \( a \) and \( b \) in Fig. 2, respectively.

this kind of photonic microstructure in an appropriate material system, the optical filtering could be adapted to the delocalization property. We believe that these results are significant towards the realization of novel optoelectronic devices based on the new concept of photonic crystals.

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